2/17/14 class 10

9.12 Information capacity of colored noise chanel

$$x(t) \rightarrow \boxed{H(f)} \rightarrow (\Sigma) \rightarrow y(t) \iff x(t) \rightarrow (\Sigma) \rightarrow \boxed{H(f)} \rightarrow y(t)$$

colored noise $n(t)$ modified colored noise $n'(t)$

$$S_{N'}(f) = \frac{S_N(f)}{|H(f)|^2}$$

Suppose N chanels

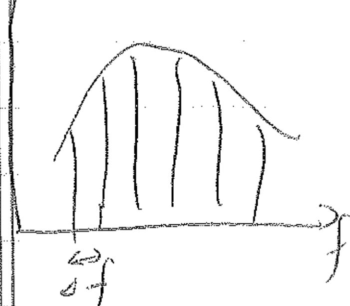
$$y_k(t) = x_k(t) + n_k(t), \quad k = 1, 2 \cdots N$$

① -- $P_k = S_X(f_k)\,\Delta f, \quad k = 1, 2 \cdots N$

② -- $\sigma_k^2 = \frac{S_N(f_k)}{|H(f_k)|^2}\Delta f \quad k = 1, 2 \cdots N$

$$C_k = \frac{1}{2}\Delta f \log_2\left(1 + \frac{P_k}{\sigma_k^2}\right), \quad k = 1, 2 \cdots N$$

$H(f)$



$$\boxed{\begin{aligned} \max\ & C = \sum_{k=1}^{N} C_k = \frac{1}{2}\sum_{k=1}^{N} \Delta f \log_2\left(1 + \frac{P_k}{\sigma_k^2}\right) \\ \text{s.t. } & \sum_{k=1}^{N} P_k = P = \text{constant} \end{aligned}}$$

lagrange multiplier λ

$$J = \frac{1}{2}\sum_{k=1}^{N} \Delta f \log_2\left(1 + \frac{P_k}{\sigma_k^2}\right) + \lambda\left(P - \sum_{k=1}^{N} P_k\right)$$

$$\frac{\partial J}{\partial P_k} = \frac{\Delta f \log_2 e}{P_k + \sigma_k^2} - \lambda = 0$$

$$P_k + \sigma_k^2 = \lambda \Delta f \log_2 e = K \Delta f \text{ constant}$$

$$= S_X(f_k)\Delta f + \left|\frac{S_N(f_k)}{H(f_k)^2}\right|\Delta f$$

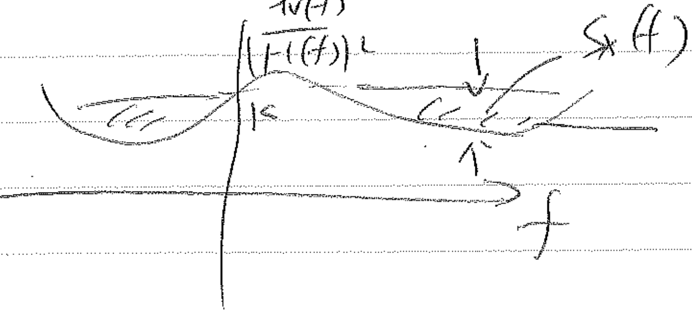
$$\Rightarrow S_X(f_k) = K - \frac{S_N(f_k)}{|H(f_k)|^2} \quad \text{for } K \geq \frac{S_N(f_k)}{|H(f_k)|^2}$$

0 otherwise

To calculate k: $P = \int \left(k - \frac{S_N(f)}{|H(f)|^2}\right) df$

$$C = \frac{1}{2} \sum_{k=1}^{N} \Delta f \log\left(k \frac{|H(f_k)|^2}{S_N(f_k)}\right) = \frac{1}{2} \int_{-\infty}^{\infty} \log_2\left(k \frac{|H(f)|^2}{S_N(f)}\right) df$$

water-filling Interpretation



shaded area is the power

Exam: give several channels, $\underbrace{\text{overall power}}_{}$ noise level
calculate the tx power for each channel to max the overall capacity

9.13 Rate Distortion Theory

rate distortion tx M-ary alphabet X_i: $1 \cdots M$
with probability P_i

R average code rate H source entropy

rx R-ary alphabet $y_j = 1 \cdots N$

$$P(X_i, y_j) = P(y_j | X_i) P(X_i)$$

~~single letter~~ average distortion $\bar{d} = \sum_{i=1}^{M} \sum_{j=1}^{N} P(X_i) P(y_j | X_i) \underline{d(X_i, y_j)}$ single letter distortion measure

$$P_D = \{ P(y_j | X_i) : \bar{d} \le D \}$$

$$I(X;Y) = \sum_{i=1}^{M} \sum_{j=1}^{N} P(X_i) P(y_j | X_i) \log\left(\frac{P(y_j|X_i)}{P(y_j)}\right)$$

rate distortion function $R(D) = \min_{P(y_j|X_i) \in P_D} I(X;Y)$

$$\text{s.t. } \sum_{j=1}^{N} P(y_j | X_i) = 1 \quad i = 1 \cdots M$$

P_i

$X_i \longrightarrow \boxed{P(y_j|X_i)} \longrightarrow y_j$

$d(X_i, y_j)$

Ex 9.14 Gaussian Source

$$d(X,y) = (X-y)^2$$

$$R(D) = \begin{cases} \frac{1}{2}\log\left(\frac{\sigma^2}{D}\right) & 0 \le D \le \sigma^2 \\ 0 & D > \sigma^2 \end{cases}$$

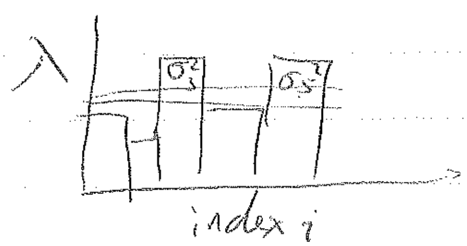
Example 9.15 set of Parral Gaussian Source

$$d = \sum_{i=1}^{N} (X_i - \hat{X}_i)^2$$

$$R(D) = \sum_{i=1}^{N} \frac{1}{2}\log\left(\frac{\sigma_i^2}{D_i}\right)$$

$$D_i = \begin{cases} \lambda & \lambda < \sigma_i \\ \sigma_i & \lambda \ge \sigma_i \end{cases}$$

λ is select s.t $\sum D_i = D$



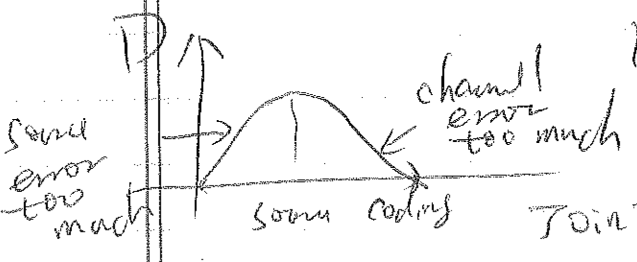
9.14 Data compression

Data is compressed and then transmitted
These two steps can be seperated
to be optimized without loss of the
optimization if there is no transmission deadline
If there is tx deadline like video conferencing

$$D = D_c + D_t$$

coding due to source

transmission due to chan



Joint source channel coding