

ECE 6332

class 5

1/29/14

exam 2/26  
coverage chapters 1-4,  
Review

electrical version  
problem

TX power (Range)

large scale propagation

path loss model:  
 $\beta = 2-6$   
log normal:  $\sigma = 3-12$  dB

small scale fading

type 1

① coherent bandwidth: RMS

flat fading v.s. frequency selective fading

② coherent time: doppler shift

slow fading v.s. fast fading

model { Rayleigh - NLOS  
complex zero mean Gaussian

Rician - LOS

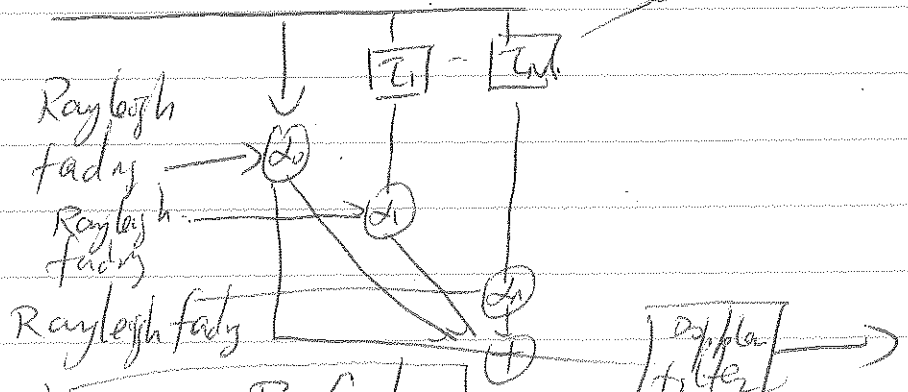
complex non zero mean Gaussian

other model: Nakagami, Gilbert, Elliot model

simulation: doppler filter

multipath profile ( $\alpha_i, \tau_i$ )

$s(t)$



# chapter 3 Statistical multipath channel models

## 3.1 Time varying channel impulse response

TX Signal  $s(t) = \text{Re}\{u(t) e^{j2\pi f_c t}\}$

$u(t)$  is baseband signal  $f_c$  is the carrier freq.

RX Signal  $r(t) = \text{Re}\left\{ \sum_{n=0}^{N(t)} d_n(t) u(t - \tau_n(t)) e^{j(2\pi f_c(t - \tau_n(t)) + \phi_{pn})} \right\}$

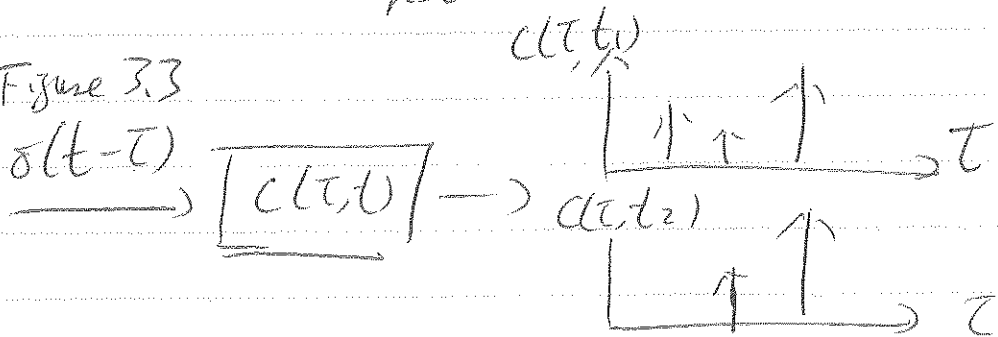
all fn of  $t$   $\left\{ \begin{array}{l} N(t) : \text{no. of multipaths} \\ d_n(t) : \text{multipath strength} \\ \tau_n(t) : \text{multipath delay} \\ \phi_{pn}(t) : \text{doppler phase shift} \end{array} \right.$

we can also write  $r(t)$  as a filter of channel

$$r(t) = \text{Re}\left\{ \int_{-\infty}^{\infty} c(\tau, t) u(t - \tau) d\tau \right\} e^{j2\pi f_c t}$$

where  $c(\tau, t) = \sum_{n=0}^{N(t)} d_n(t) e^{-j2\pi f_c \tau_n(t) - \phi_{pn}} \delta(\tau - \tau_n(t))$

Figure 3.3



~~Ex 3.1~~ If channel time invariant

$$c(\tau) = \sum_{n=0}^N d_n e^{-j\phi_{pn}} \delta(\tau - \tau_n)$$

Ex 3.1

### 3.2 Narrowband Fading models

i.e. flat fading  $\rightarrow$  RMS small

$$r(t) = \text{Re} \left\{ u(t) e^{j2\pi f_c t} \left( \sum_n d_n(t) e^{-j\phi_n(t)} \right) \right\}$$

assume  $u(t) = e^{j\phi_0}$ ,  $\phi_0$  random

$$s(t) = \text{Re} \left\{ u(t) e^{j2\pi f_c t} \right\} = \text{Re} \left\{ e^{j(2\pi f_c t + \phi_0)} \right\}$$

$$\text{then } r(t) = \text{Re} \left\{ \sum_{n=1}^N d_n(t) e^{-j\phi_n(t)} e^{j2\pi f_c t} \right\}$$

$$= r_I(t) \cos(2\pi f_c t) - r_Q(t) \sin(2\pi f_c t)$$

where

$$r_I(t) = \sum_{n=1}^N d_n(t) \cos \phi_n(t) \quad \text{In phase}$$

$$r_Q(t) = \sum_{n=1}^N d_n(t) \sin \phi_n(t) \quad \text{Quadrature phase}$$

$$\phi_n(t) = 2\pi f_c \tau_n(t) - \phi_{0n} - \phi_0$$

Question: what is power spectrum density (PSD)  
What is distribution

#### 3.2.1 Auto correlation, cross-correlation and PSD

mean:  $E[r_I(t)] = E \left[ \sum_n d_n \cos \phi_n(t) \right] = \sum_n E[d_n] E[\cos \phi_n(t)] = 0$

cross correlation:  $E[r_I(t) r_Q(t)] = 0$

Auto correlation  $A_{r_I}(t, t+\tau) = E[r_I(t) r_I(t+\tau)]$

$$= \frac{1}{2} E \left[ \sum_n d_n^2 \right] \cos(2\pi \nu \tau \cos \frac{\theta_n}{\lambda})$$

if  $f_{\nu n} = \nu \cos \theta_n / \lambda$  is fixed  $\theta_n$  not function of  $t$   
it is wide sense stationary  $A_{r_I}(\tau)$

~~cross correlation~~ for received signal

$$r(t) = r_I(t) \cos(2\pi f_c t) - r_Q(t) \sin(2\pi f_c t)$$

$$A_r(\tau) = E[r(t) r(t+\tau)]$$

$$= A_{r_I}(\tau) \cos(2\pi f_c \tau) + A_{r_I, r_Q}(\tau) \sin(2\pi f_c \tau)$$

if we assume  $\Delta f = 2\pi / N$ ,  $E[d_n^2] = 2P_r / N$

$$A_{r_I}(\tau) = P_r J_0(2\pi \tau / \Delta t) \quad \text{Figure 3.5}$$

$$A_{r_I, r_Q}(\tau) = 0$$

PSD  $S_{r_c}(f) = S_{r_d}(f) = FVA_{r_c}(f) = \frac{2P_r}{\pi f_0} \frac{1}{\sqrt{1-(f/f_0)^2}}$

Figure 3.6

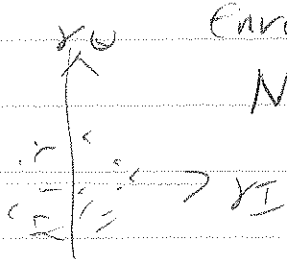
$f \leq f_0$   
0 otherwise

### 3.3.2 Envelope and Power distributions

envelop  $r(t) = |x(t)| = \sqrt{r_I^2(t) + r_Q^2(t)}$

NLOS:

zero mean complex Gaussian  $r_I + j r_Q$



$$P_r(r) = \frac{r}{\sigma^2} \exp\left(-\frac{r^2}{2\sigma^2}\right), r \geq 0$$

Rayleigh distribution with rx power  $\sigma^2$

example 3.2

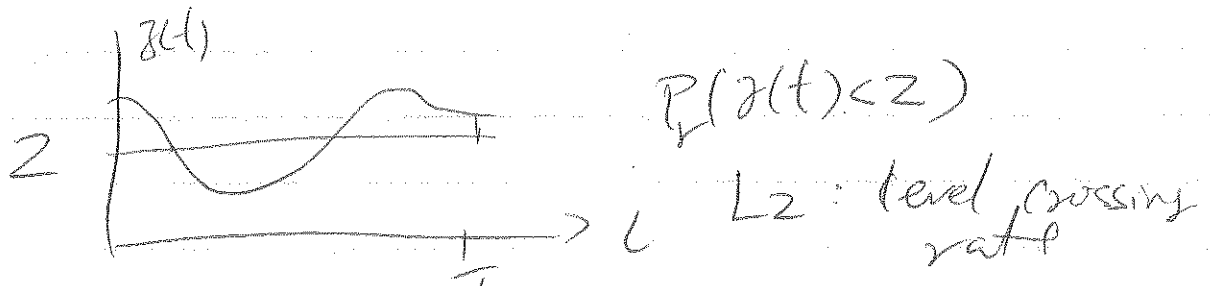
LOS: non-zero mean  $(\mu)$  complex Gaussian  $r_I + j r_Q$

Rician distribution  $P_r(r) = \frac{r}{\sigma^2} \exp\left(\frac{-r^2 + \mu^2}{2\sigma^2}\right) I_0\left(\frac{r\mu}{\sigma^2}\right)$

$r > 0$

Nakagami distribution (m)

### 3.2.3 level crossing rate and average fade duration



### 3.2.4 Finite state Markov channel

