Stability analysis of Inflation and extension of isotropic, homogeneous, thick-walled cylinders

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The inflation and extension of cylindrical membranes has received much attention in the last few decades.The study of thick-walled structures is far from complete due to numerous difficulties associated with the analysis. The major obstacle has been that the governing equilibrium equations for thick-walled structures, even for axisymmetric deformations, are non-linear partial dfifferential equations. We seek to understand the stability of cylindrical solutions for non-linear,homogeneous and isotropic materials.We use a variational approach to derive the equations of equilibrium for axisymmetric deformations by minimizing the total potential energy of the cylindrical body inflated by an incompressible fluid. In general closed-form analytic solutions are unlikely to exist, so we obtain a first integral of the equilibrium equations that would give us an insight into the behaviour of solutions. In some particular cases, cylindrical solutions for certain classes of materials can be written explicitly.

We assess the stability of cylindrical deformations, without assuming special forms of constitutive relations, from the second variation of energy functional or by linearizing the equilibrium equations about cylindrical deformations. The inhomogeneous deformation field for this boundary value problem makes it difficult to derive algebraically explicit stability criteria in the form of necessary and sufficient conditions. We demonstrate the existence of nontrivial solutions to the linearized equations, which is a necessary condition for bifurcation to noncylindrical deformations, for a class of compressible materials, i.e., harmonic materials. We obtain explicit closed-form solutions of the linearized equations for harmonic materials. For specific forms of strain energy function(s) of harmonic materials, we explicitly calculate the loading conditions at which this necessary condition is satisfied. This work is an important step towards the development of a complete three-dimensional theory of the mechanics of cylindrical structures.

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