Linear Parameter-Varying Lean Burn Air-Fuel Ratio Control for a Spark Ignition Engine

Maximization of the fuel economy of the lean burn spark ignition (SI) engine strongly depends on precise air-fuel ratio control. A great challenge associated with the air-fuel ratio feedback control is the large variable time delay in the exhaust system. In this paper, a systematic development of an air-fuel ratio controller based on post lean NOx trap (LNT) oxygen sensor feedback using linear parameter-varying (LPV) control is presented. Satisfactory stability and disturbance rejection performance is obtained in the face of the variable time delay. The LPV controller is simplified to an explicit parameterized gain scheduled lead-lag controller form for the ease of implementation. A Ford F-150 truck with a V8 4.6 l lean burn engine was used to demonstrate the LPV air-fuel ratio control design. Both simulation and experimental results demonstrate that the designed controller regulates the tailpipe air-fuel ratio to the preset reference for the full engine operating range. [DOI: 10.1115/1.2745849]

1 Introduction

In 2003, U.S. consumed about 20 million barrels of oil per day. The gasoline for cars and light trucks accounts for 45% of the total oil consumption. Lean burn technology for gasoline engines has drawn great attention during the past decade, largely due to its potential for improving fuel economy and reducing CO₂ emissions [1]. A lean burn engine is designed to operate at high intake manifold pressure with an air-fuel ratio greater than 10 and less than 23. Consequently, combustion efficiency can be improved through reduced pumping losses and enhanced thermodynamic efficiency. Compared to the conventional port fuel injection (PFI) engine, the gasoline lean burn engine presents a new set of challenges to the engine control community. The main challenge for lean burn technology is that, under lean operating conditions, the conventional three-way catalyst (TWC) system is no longer effective in reducing NOx pollutants. A special TWC with NOx trapping and conversion capabilities, known as lean NOx trap (LNT), has to be used downstream of the conventional TWC to meet the government emission standards. During the lean operation, NOx in the feed gas is stored in the LNT. When the stored NOx reaches a certain threshold, the trap must be purged by switching to rich operation for a short period of time to regenerate the storage capacity and recover the efficiency. The NOx released from the LNT during the purge period is converted into non-polluting nitrogen by the rich air-fuel mixture [2–5].

Properly managing the storage and purge cycles is critical for achieving the fuel economy and NOx emission control targets of the lean burn gasoline engine. The desired tailpipe air-fuel ratio profile (reference air-fuel ratio) is defined by the LNT purge control [6,7], with the objectives of optimizing fuel economy while satisfying emission constraints. Therefore, it is necessary to design a controller to regulate the tailpipe air-fuel ratio to follow the air-fuel reference for both the NOx storage phase (lean operation) and the purge phase (rich operation) in order to accomplish the LNT purge control.

In this paper, we concentrate on the air-fuel ratio control for the storage phase, that is, the design of the “outer-feedback loop” air-fuel ratio controller is considered. A linear universal exhaust gas oxygen (UEGO) sensor is used downstream of the LNT to measure the tailpipe air-fuel ratio. The air-fuel ratio controller to be designed is used to generate the commanded air-fuel ratio for the fuel injection system. During the storage phase when the engine is operating under lean conditions, the air-fuel ratio is selected to (i) meet the driver’s demand, (ii) maximize fuel economy, and (iii) satisfy other constraints, such as lean burn limit [7]. These requirements dictate the set-point selection, and the optimal choice for the air-fuel ratio in the storage phase is usually a constant set-point for steady state operation.

A number of publications have described various designs of air-fuel ratio controllers for the stoichiometric feedgas air-fuel control [8–11]. Related work in the air-fuel control for lean burn is limited. In [12], an adaptive-feedforward model-based feed gas air-fuel ratio controller was developed for a 4 cylinder 2.2 l Mercedes-Benz lean burn engine. The control performance is largely dependent on the control-oriented engine model. However, it is very difficult to establish an accurate “outer-feedback loop” emission model covering all operating conditions over the engine life cycle for systems involving TWC and LNT. In addition, there is significant open-loop uncertainty in the fuel injection and exhaust system, such as canister purge, which cannot be handled by the feedforward control. Therefore, feedback control is necessary in order to maintain accurate air-fuel ratio control.

The biggest challenge associated with the air-fuel ratio feedback control stems from the variable time delay in the exhaust system. Since the UEGO sensor is positioned after the LNT, a significant time delay occurs between the UEGO sensor signal and the effective change of the engine feedgas air-fuel ratio. In addition, the time delay is largely dependent on the engine operating condition defined by the engine speed and the air mass flow. Throughout the engine operating envelop, the time delay can change significantly due to the space velocity variation. For a constant delay that can be measured or estimated precisely, the Smith predictor is an effective dominant delay compensator [13,14] and \( H_\infty \) control of delay systems based on the modified Smith predictor [15,16] has been proposed recently. However, these results cannot be applied to delay systems with a variable delay as in the proposed air-fuel ratio control problem. For example, according to the experimental vehicle data collected on a Ford truck, the delay in this engine varied from 0.3 s to 2.7 s. Such a large variable time delay is the main hurdle to achieve the desired disturbance rejection performance using a single control-
Fig. 1 Aftertreatment system and “outer-feedback” loop AFR control system configuration in the lean burn mode

For the full engine operating range, therefore, the variability of the time delay must be taken into account in the control design so that the desired performance can be obtained for the full engine operating envelope.

Model predictive control (MPC) has been proposed and applied to a wide range of application areas [17]. Current MPC technology offers significant new capabilities, but several limitations still remain. The algorithms are not nominally stabilizing, so that tuning choices must be tested through extensive closed-loop simulation [17]. In general, the corresponding dynamic optimization problem is solved on-line at each control execution limiting its applications to slow processes.

Linear parameter varying (LPV) gain-scheduling control [18–20] has recently received significant attention because it provides a systematic way of computing gain scheduled controllers for nonlinear and parameter dependent systems with stability and performance guarantees. Numerous applications of LPV controllers can be found in various areas, such as, flight control [21,22], engine control [23,24], and vibration isolation [25]. In this work, we will use an LPV gain-scheduling approach to design the air-fuel ratio controller. The scheduling parameter will be the time delay in the exhaust system, which is considered to be a function of the engine operating point. The relation between the time delay and the engine operating point that defines the time delay and the engine operating point is defined by the engine speed and air flow mass is identified off-line. On-line identification of the time delay in the exhaust system can be found in [26] for stoichiometric burn engines. However, on-line identification is slow and gain-scheduling based on the on-line identification is prohibitive. Hence, in the present work a parametric expression is used to estimate the delay based on measurements of engine speed and air mass flow. The experimental vehicle data used for the time delay identification were collected from a Ford F-150 truck with a V8 4.6 l lean burn engine. The designed LPV controller was tested on the same truck model. All the experimental data presented in the paper were provided by Ford Motor Company.

The paper is organized as follows: Sec. 2 describes the engine model for the air-fuel ratio subsystem in the lean burn mode. The time delay in the exhaust system is identified and a simplified engine model for control design is proposed. In Sec. 3, the LPV controller design for the simplified engine model is presented. The LPV controller discretization and implementation are discussed in Sec. 4. Finally, the experimental results are presented in Sec. 5. Section 6 concludes the paper.

2 Engine Model for Air-Fuel Ratio Dynamics in the Lean Burn Mode

The lean burn aftertreatment system with commonly used sensors and the “outer-feedback” control configuration is shown in Fig. 1. It consists of a conventional TWC, an LNT, a heated exhaust gas oxygen (HEGO) sensor downstream of the TWC, and two UEGO sensors downstream of the engine and LNT. According to hardware assumptions on available and future vehicle platforms, the upstream UEGO sensor measurement will not be available for the air-fuel ratio control. Therefore, the measurement signal of the controller is the air-fuel ratio measured by the UEGO sensor downstream of the LNT. The controller regulates the air-fuel ratio of the air-fuel mixture entering the engine to follow the reference air-fuel ratio using engine measurements, such as air mass flow and engine speed. The commanded fuel for the fuel injector is calculated based on the air-fuel ratio command and the cylinder air charge per engine cycle. Thus, the input of the open loop engine model is the commanded air-fuel ratio at the fuel injector and the output is the measured air-fuel ratio by the post-LNT UEGO sensor. It is noted that fuel pooling in the intake manifold is compensated in the fuel injection system using a production fueling control strategy as it is often the case in practice.

During the lean burn mode, the TWC eventually becomes saturated by O2, therefore, the complex oxygen dynamics introduced by the TWC and the LNT can be approximated by a pure transport time delay. Because of the UEGO sensor location, the transport time delay is significantly greater than the time constant of the UEGO sensor. Due to the feedback control bandwidth limitation introduced by the time delay, the dynamics of the UEGO sensor is above the control bandwidth and can be neglected for feedback control design. Hence, the simplified engine “outer-feedback loop” air-fuel ratio model can be considered as a pure variable time delay, which can be identified using experimental vehicle data. A first-order ARX model and time domain least squares estimation method is used for model identification. Figure 2 shows a representative experimental and identified response. The solid line is the measured air-fuel ratio and the dashed-dotted line is the estimated model output. The mean air-fuel ratio is removed from the data for identification purposes. It can be seen that the identified model matches the measurements data well.

The identified overall time delay \( \tau \) consists mainly of two parts: the cycle delay \( \tau_c \) and the exhaust gas transport delay \( \tau_g \). The cycle delay \( \tau_c \) is due to the four stokes of the engine and it is approximately one engine cycle. Hence, the cycle delay is given by

\[
\tau_c = \frac{720}{(360/60)n} = \frac{120}{n} \text{ (s)}
\]

where \( n \) is the engine speed in RPM. The transport delay \( \tau_g \) is due to the exhaust gas flowing from the exhaust valve to the tailpipe UEGO sensor and it varies inversely with the air flow rate assuming an average exhaust temperature, i.e.,

\[
\tau_g = \frac{\alpha}{m_a} \text{ (s)}
\]

where the coefficient \( \alpha \) is determined based on experimental data and \( m_a \) is the air mass flow in lb/min. Thus, an approximate estimate for the overall time delay of the feedback system is given by

\[
\tau = \tau_c + \tau_g.
\]

Since one engine cycle delay has a fixed relation with the engine speed, we will remove the engine cycle delay part from the identified time delay. The residue is the exhaust delay which is
considered to be solely dependent on the air mass flow. Based on the time delay identification results, we obtain an average value \( \alpha = 1.831 \) for the F-150 V8 4.6 l engine. Hence, the approximate estimation of the time delay is given by

\[
\tau = \left[ \frac{120}{n} + \frac{1.831}{\dot{m}_a} \right] \text{s}
\]  

(1)

Figure 3 shows the relation between the air mass flow and the exhaust delay. The data points represent the identified exhaust delay \( \tau - \tau_c \) and the solid line is the estimated exhaust delay \( \tau_e = \alpha/\dot{m}_a \). The average estimation error is 0.115 s. It is worth mentioning that the sampling time for the experimental data is 0.1 s.

Therefore, the delay estimation using Eq. (1) approximates the identified time delay very well. In the following, the obtained time delay estimation is used to schedule the designed LPV controller. Generally, mass air flow can be determined based on the speed density relationship or based on flow sensor measurements. Due to hardware limitations on the available vehicle platform, measurements required for the speed density calculation are not available. In the present work, the mass air flow \( \dot{m}_a \) is measured using a production hot wire mass air flow sensor.

3 LPV Controller Design for a Simplified Engine Model

3.1 Controller Design Objectives and Challenges. During an NOx purge cycle, the engine is running at a rich air-fuel ratio. Then, the engine is subsequently commanded to run at a predetermined lean air-fuel ratio, typically 1.1 or 1.4 times the nominal stoichiometry. A feedback controller is required to regulate the lean air-fuel ratio in the presence of disturbances, such as, canister purge, open loop uncertainties, and unmodelled dynamic effects. The design objectives include the following:

(a) Tracking performance: The controller should control the engine to reach the reference air-fuel ratio as quickly as possible and the final steady state error should be zero. It is required that the air-fuel ratio regulation performance remains the same for different engine operating conditions for drivability and aftertreatment purposes.

(b) Disturbance rejection: For the system subject to uncertain disturbances, such as canister purge and measurement noise, the air-fuel ratio excursion should be as low as possible.

(c) Transient response: The percentage overshoot should be minimized. 0% overshoot is desired. There is a trade-off between this requirement and the disturbance rejection requirement.
(d) Implementation: The designed controller should have a simple structure for the ease of implementation and calibration.

Furthermore, the above design objectives need to be achieved for the full engine operating envelope. From Sec. 1, it can be seen that the variation of the time delay due to the engine operating condition change is significant. It is known that the time delay results in a bandwidth limitation for the feedback control. Thus, the main challenge is the present work is to handle the large time delay variation so that satisfactory feedback control performance is achieved regardless of the delay. In addition, the controller design method should be systematic and generic, i.e., the same design procedure should be carried out to obtain controllers for different models of the vehicles.

3.2 LPV Control Design Interconnection. The general system interconnection for the LPV controller design in the lean burn mode is shown in Fig. 4, where \( r(t) \) is the commanded air-fuel ratio input for the fuel injection system, \( y(t) \) is the tailpipe air-fuel ratio measurement, and \( G_{\text{sys}} \) is the simplified engine model, which is considered to be a pure variable time delay. Similar engine modeling simplification strategies can be found in [27]. For controller synthesis, \( G_{\text{sys}} \) is approximated by a first order parameter dependent Padé approximation given by

\[
G_{\text{sys}}(s) = \frac{1 - \frac{1}{2} \tau s}{1 + \frac{1}{2} \tau s}
\]

where \( \tau \) is the variable time delay in the engine model. Since we are addressing set-point air-fuel regulation during steady-state lean burn operation, the delay remains constant at the specific operating condition. Higher order Padé approximation can be used for the LPV design at the cost of increasing the order of the designed controller, which is undesirable for implementation. In order to achieve zero steady state error corresponding to an air-fuel ratio set-point, the LPV controller is selected to have an ex-order to achieve zero steady state error corresponding to an air-fuel ratio set-point, the LPV controller is selected to have an ex-order to achieve zero steady state error corresponding to an air-fuel ratio set-point, the LPV controller is selected to have an ex-order to achieve zero steady state error corresponding to an air-fuel ratio set-point, the LPV controller is selected to have an ex-order to achieve zero steady state error corresponding to an air-fuel ratio set-point, the LPV controller is selected to have an ex-order to achieve zero steady state error corresponding to an air-fuel ratio set-point, the LPV controller is selected to have an ex-order to achieve zero steady state error corresponding to an air-fuel ratio set-point, the LPV controller is selected to have an ex-order to achieve zero steady state error corresponding to an air-fuel ratio set-point, the LPV controller is selected to have an ex-order to achieve zero steady state error corresponding to an air-fuel ratio set-point, the LPV controller is selected to have an ex-order to achieve zero steady state error corresponding to an air-fuel ratio set-point, the LPV controller is selected to have an ex-order to achieve zero steady state error corresponding to an air-fuel ratio set-point, the LPV controller is selected to have an ex-order to achieve zero steady state error corresponding to an air-fuel ratio set-point, the LPV controller is selected to have an ex-order to achieve zero steady state error corresponding to an air-fuel ratio set-point, the LPV controller is selected to have an ex-order to achieve zero steady state error corresponding to an air-fuel ratio set-point, the LPV controller is selected to have an ex-order to achieve zero steady state error corresponding to an air-fuel ratio set-point, the LPV controller is selected to have an ex-order to achieve zero steady state error corresponding to an air-fuel ratio set-point, the LPV controller is selected to have an ex-order to achieve zero steady state error corresponding to an air-fuel ratio set-point, the LPV controller is selected to have an ex-order to achieve zero steady state error corresponding to an air-fuel ratio set-point, the LPV controller is selected to have an ex-order to achieve zero steady state error corresponding to an air-fuel ratio set-point, the LPV controller is selected to have an ex-order to achieve zero steady state error corresponding to an air-fuel ratio set-point, the LPV controller is selected to have an ex-order to achieve zero steady state error corresponding to an air-fuel ratio set-point, the LPV controller is selected to have an ex-order to achieve zero steady state error corresponding to an air-fuel ratio set-point, the LPV controller is selected to have an ex-order to achieve zero steady state error corresponding to an air-fuel ratio set-point.

3.3 Performance Weight Selection. The weighting functions play an important role in the LPV control design. For improved performance, high order weighting functions can be applied. However, this will increase the order of the dynamic controller. Thus, for our purposes a constant performance weight and a first order robustness weight are used in the present application. The selection of these weights is discussed below. Generally, there is no explicit solution for the weight selection which will lead to an optimal design. Therefore, the desired performance is obtained by carefully tuning the corresponding weighting functions. In automotive applications, a simplified and systematic weight selection process is important for testing and production purposes. Then, controllers can be easily designed for different models of a vehicle. As a result, a systematic weight function selection and tuning procedure is desirable for the proposed LPV air-fuel ratio control design. In the following, a generic weighting function selection procedure is developed for the lean burn air fuel ratio control. The selected weighting functions are parameterized by the time delay and the desired performance can be easily obtained by tuning a so called “aggressive factor” in the formulation of the weighting function.

Consider the weighted control structure in Fig. 5, where \( r \) is the reference signal, \( y \) is the output, and \( G_{\text{lp}}(s) \) is a first order Padé approximation of the time delay. Therefore, the loop transfer function is given by

\[
L(s) = G_{\text{lp}}(s)K(s)
\]

where \( W_p \) is the performance weight on the sensitivity function \( S = 1/(1+L) \) and \( W_T \) is the robustness weight on the complementary sensitivity function \( T = L/(1+L) \). Consider the following selection of weighting functions:

\[
W_p(s) = \frac{\omega_B^*}{s}
\]

\[
W_T(s) = \frac{s}{s/M + \omega_{BT}^*}
\]

The weight \( W_p(s) \) specifies a minimum bandwidth \( \omega_B^* \) (actually \( \omega_B^* \) is the frequency where the straight-line approximation of the weight crosses 0 dB) and zero steady-state error. It also specifies a sensitivity function \( S(j\omega) \) whose magnitude increases by 20 dB/decade at frequencies lower than the bandwidth when the condition \( |W_p(j\omega)S(j\omega)| < 1 \) is satisfied. Therefore, the bandwidth requirement of the closed-loop system can be used for the selection of \( \omega_B^* \). The selection of the robustness weight \( W_T(s) \) is based on the phase margin requirement. As a rule-of-thumb, a percentage overshoot condition on the closed-loop system will be approximated as a phase margin constraint. Therefore, in order to obtain robust stability for a multiplicative delay uncertainty \( e^{\pm\theta} \), \( \omega_{BT}^* \) is selected such that \( \omega_{BT} < 1/\theta \). Since the overshoot is related to the phase margin which is another index of robustness, we will investigate the connection between phase margin and the uncertainty weight \( W_T(s) \).

It is known that a time delay \( \theta \) yields an additional phase contribution of \( -\theta \omega, \) \( (-57 \theta \omega, \text{deg}) \), where \( \omega \) is the gain crossover frequency. Therefore, a phase margin \( P_m \) (deg) indicates that the closed-loop system is robustly stable for a delay uncertainty \( \theta = P_m/57 \omega \). On the other hand, if the system is robustly stable for a delay uncertainty \( \theta \), then the system will have a phase margin at least \( P_m = 57 \omega \). Therefore, the \( \theta \) value in the robustness weight \( W_T \) can be tuned to achieve the desired phase margin of the system. As the \( \theta \) value increases, \( \omega_{BT} \) decreases. Consequently, the bandwidth of the closed-loop system after design decreases and the corresponding phase margin increases and vice versa. Hence, the desired phase margin and step response can be achieved by tuning the value of \( \theta \). Thus, \( \theta \) is named “aggressive factor” of the weight (4). Furthermore, an appropriate \( \theta \) value to start the design can be chosen as

\[
\theta = \frac{P_m \tau}{57}
\]

where \( P_m \) is the phase margin requirement. Due to the bandwidth limitation imposed by the time delay, the upper bound of the gain crossover frequency \( \omega_c \) using feedback control is less than \( 1/\tau \) rad/s. As a result, the system will have a minimum phase
margin $P_m$ (deg) if the condition \( \left\| W_T(j\omega)T(j\omega) \right\| < 1 \) is satisfied.

Now, back to the design interconnection in Fig. 4, it can be observed that an integrator is introduced explicitly in the control loop. Therefore, the integrator in the weight $W_P$ can be removed and the corresponding performance weights can be selected as follows:

\[
W_1 = \omega_p^s
\]

\[
W_2 = \frac{s}{s/M + 1/\theta}
\]

It is desired for the closed-loop system to have a bandwidth larger than 0.1 rad/s. Therefore, the performance weight is chosen as $W_1 = 0.1$

In the above discussion, the selection of $\theta$ is based on the phase margin requirement. In general, a phase margin larger than 65 deg yields a percentage overshoot less than 3\%. Therefore, the phase margin requirement is selected as $P_m \geq 65$ deg. For the LPV design, the “aggressive factor” $\theta$ is tuned to provide the best overall performance as long as $M > 1$. A value of $M=300$ is selected in Eq. (7). This value does not significantly affect the controller performance. Hence, the proposed tuning procedure is the following:

**Step 1:** Set $P_m=65$ deg, $M=300$, and $\omega_p=0.1$.

**Step 2:** Calculate $\theta$ using Eq. (5), where $\tau$ is the maximum time delay considered for the LPV controller design.

**Step 3:** Select the weighting functions as in Eq. (6) and Eq. (7).

**Step 4:** Design the LPV controller according to the control structure in Fig. 4.

**Step 5:** Check the step responses. If the overshoot is observed, increase $\theta$ and go back to step 2. If the response is sluggish, decrease $\theta$ and go back to step 2.

**Step 6:** Obtain the final LPV controller.

The selection of $\theta$ for our LPV design is $\theta=1.4$ and the corresponding robustness weight is given by

\[
W_2 = \frac{s}{s/300 + 1/1.4}
\]

Details on the LPV controller synthesis are provided in the following section.

### 3.4 LPV Controller Synthesis

The state space realization of the first order Padé approximation Eq. (2) is

\[
\begin{align*}
\dot{x} &= -\frac{2}{\tau} x + u \\
y &= \frac{4}{\tau} x - u
\end{align*}
\]

It can be seen that the plant Eq. (8) is parameterized by $\rho=1/\tau$ affinely. Therefore, for the LPV controller design, $\rho$ is chosen as the scheduling parameter and will be used to schedule the LPV controller in real time. This parameter can be calculated based on the measured engine speed and the air mass flow using Eq. (1). The augmented control design interconnection in Fig. 4 has three state variables. The synthesis of the proposed LPV controller involves the solution of a set of three parameter-dependent linear matrix inequalities (LMIs) [18,19]. These inequalities can be reduced to a set of finite dimensional LMIs, first by appropriate selection of basis functions that define the functional dependence of the Lyapunov matrices $X(\rho)$ and $Y(\rho)$ on $\rho$, and then by gridding the parameter space and solving the inequalities at the grid points [18,28,29]. However, for validation the constraints have to be checked later on a much denser grid. For the problem at hand, we select the following form for the Lyapunov parameter matrices,

\[
X(\rho) = X_0
\]
\[ Y(\rho) = Y_0 + \rho Y_1 + \frac{\rho^2}{2} Y_2 + \frac{\rho^3}{3} Y_3 \]

The choice \( X(\rho) = X_0 \), a constant Lyapunov matrix, allows the LPV controller to be designed so that it is rate independent [18]. In other words, the LPV controller is not a function of the rate of variation of the parameter allowing easy practical implementation.

According to the time delay identification results, the maximum time delay is 2.7 s when the engine is idling and the minimum time delay is 0.3 s. However, instead of using the full delay variation for the LPV controller design, a time delay range \([0.7 \ 2.7]\) s will be considered for two reasons. First, the parameter for LPV controller synthesis is the inverse of the time delay \( \rho = 1/\tau \). Thus, expanding the design parameter range from \([0.7 \ 2.7]\) s to \([0.3 \ 2.7]\) s enlarges the synthesis parameter range of \( \rho \) significantly, and increases the conservatism of the LPV control design. Second, the vertex controller designed for a 0.7 s delay can be used to accommodate all the possible delays less than 0.7 s. Such an accommodation will not result in any stability concern, because it is equivalent to time delay overestimation. The phase margin of the closed-loop system increases when the time delay is overestimated.

The rate of variation of the parameter \( \rho \) is assumed to lie in the set \([-0.5 \ 0.5]\) for the LPV controller design. As \( \rho \) appears linearly in the LMIIs, we only need to check the corresponding LMIIs at the extreme points -0.5 and 0.5. Designing rate-independent controllers does not mean that the rate information is not used during the design. The LPV controllers are guaranteed to provide the desired stability and performance when the rate bounds on the parameter variations are respected. The design parameter grid is spaced uniformly with 21 points, i.e., there is a grid point every 0.1 s from 0.7 s to 2.7 s. The achieved induced \( L_2 \) norm performance level is 0.585. The LMI solutions are further checked on a 201 points grid, i.e., using a grid point every 0.001 s. The designed LPV controller consists of 21 vertex controllers. Linear interpolation is used to obtain controllers when the corresponding parameters are not exactly on the grid points.

The designed LPV controller provides satisfactory time-domain and frequency domain performance for the full engine speed operating region. The designed controller is in the standard LPV form [18]. However, this controller has a complicated structure that does not allow its implementation in automotive applications. In the next section, we will reduce the controller order and we will formulate the scheduled controller in a simple analytical parameter-dependent format with coefficients represented as functions of the time delay, which greatly simplifies its practical implementation.

### 3.5 Controller Order Reduction and Explicit Parameterization

Each vertex controller of the designed LPV controller has 3 poles and 2 zeros. However, there is only 1 dominant pole and 1 dominant zero for every vertex controller. The additional 2 poles and 1 zero only characterize the high frequency dynamics of the controller outside the frequency region of interest. A simplified controller structure can be achieved by removing the high frequency dynamics of the vertex controllers and keeping only the corresponding steady state gain. Figure 6 shows the LPV controller model reduction results. It can be seen that the low frequency characteristics of the controller are preserved very well.

As a result, the scheduled controller obtains the following first order lead-lag parameter dependent structure:

\[
K(s) = K_p \frac{s + 1}{T_1 + 1} \frac{s + 1}{T_2 + 1} \frac{s + 1}{T_3 + 1} \tag{9}
\]

where the parameters \( K_p \), \( T_1 \), \( T_2 \), and \( T_3 \) are functions of the time delay, which is a function of the engine speed \( n \) and the air mass flow \( m_a \). To simplify the controller implementation, we obtain analytical expressions of the parameters \( K_p \), \( T_1 \), and \( T_2 \) as functions of the delay using polynomial interpolation (see Fig. 7). The results of the polynomial fitting are the following:

\[
K_p = 0.1807 \left( \frac{1}{\tau} \right)^3 - 0.6701 \left( \frac{1}{\tau} \right)^2 + 0.8173 \left( \frac{1}{\tau} \right) - 0.0133
\]

\[
T_1 = 2.0010 \left( \frac{1}{\tau} \right) - 0.0005
\]

\[
T_2 = 0.1030 \tau + 0.6736 \tau^2 + 1.5683 \tau + 0.1984
\]

The expressions in (10) provide a simple functional dependence of the parameters \( K_p \), \( T_1 \), and \( T_2 \) with respect to the delay \( \tau \) that define a simple explicit parameterization of the LPV controller. Such an explicit functional dependence cannot be obtained from single point \( H_\infty \) designs.

The LPV controller design does not use any vehicle information other than the time delay estimation result and the design is solely based on the time delay range. This results in a general implementation framework because the designed controller can be implemented on any other vehicle with the same time delay range without any controller redesign. However, the time delay estimation used for scheduling needs to be tailored for different vehicles.

### 3.6 Controller Validation

The explicit simplified gain scheduled controller is validated via frequency response analysis and time-domain simulations. Figure 8 shows the Bode diagrams of the loop transfer functions. It can be seen that all loop transfer functions cross over 0 dB between 0.2 rad/s and 0.3 rad/s. In addition, the loop transfer functions roll off -20 dB/decade at frequencies lower than the bandwidth. Clearly, the designed loop transfer functions satisfy the design requirements. Figure 9 shows the Nichols charts of the loop transfer functions. It can be seen that all loop transfer functions have a phase margin approximately 65 deg. Also, all the Nichols charts are tangent to the 0 dB circle, which indicates that the percentage overshoot should be close to 0. Figure 10(a) shows the step responses at the design grid points using the exact delay representation. The lower time delay limit of the controller design is 0.7 s, therefore the vertex controller designed for a 0.7 s delay is used to accommodate all the possible delays less than 0.7 s. Figure 10(b) shows the step responses for time delays less than 0.7 s. It can be seen there is no significant effect using the vertex controller at 0.7 s. Furthermore, it can be observed that the settling time is less than 10 s and the percentage overshoot is less than 2% for all the design grid points. Hence, the
designed LPV controller satisfies the design objectives. On the other hand, neglecting the time delay variation and designing a single nominal $H_\infty$ controller for the longest delay case (2.7 s) results in poor tracking performance for shorter delays (see Fig. 11). This result demonstrates the importance of taking into account the delay variability in the control design.

![Bode Diagram](image)

**Fig. 8** Bode diagrams of the loop transfer functions

![Nichols Chart](image)

**Fig. 9** Nichols charts of the loop transfer functions
4 LPV Controller Implementation and Validation

For implementation purposes, the LPV controller needs to be discretized. The sampling rate for discretization is chosen as 50 ms according to the setting of the Ford air-fuel control algorithm. The approach in [30] is used to discretize the continuous-time LPV controller.

The proposed LPV controller implementation block diagram is shown in Fig. 12. The delay estimation Eq. (1) is developed based on steady state engine operation conditions. During the transient engine operation, the cycle delay is varying simultaneously with the engine speed variation, but, the exhaust transport delay does not vary as fast as the air mass flow varies. The high frequency content in the air flow rate measurement will not immediately contribute to the time delay variation due to the relative slow thermal and fluid dynamics in the exhaust gas. Therefore, in order to remove the high frequency content in the air mass flow measurement, a discrete-time low pass filter given by

\[ x(n) = 0.9x(n-1) + 0.1u(n-1) \]
is applied to the air mass flow measurement before the delay estimation. The saturation block is used to bound the estimated time delay in its designated range.

### 4.1 Simulation Results

The designed controller implementation strategy is validated through time-domain simulations and experimental vehicle tests. An experimental speed and air mass flow profile from an FTP drive circle (see Fig. 13) is used for simulation. Figure 14 shows the corresponding estimated delay profile in the engine. The engine model for the simulation is a variable time delay component that uses the estimated time delay profile. In order to characterize the open loop disturbances in the engine operation, such as fuel injector uncertainty and canister purge, an alternating pulse disturbance profile of time duration 25 s and amplitude ranging from −0.02 to 0.02 is added to the air-fuel ratio input of the engine. The purpose of the simulation is to investigate the disturbance rejection performance of the designed LPV controller at different engine operating conditions.

In practice, the estimated time delay will not exactly match the actual time delay in the engine and exhaust system. Therefore, simulations with delay estimation errors have also been carried out. Figure 15 shows the simulation results for 3 cases: (i) exact time delay estimation, i.e., the LPV controller uses the same time delay as the actual time delay in the engine model; (ii) 20% time delay underestimation; (iii) 20% time delay overestimation. The dashed line is the preset air-fuel reference signal. The solid line is the simulated tailpipe air-fuel ratio output when the estimation of the time delay is exact. The corresponding underestimated delay case is shown by the dashed-dotted line and the overestimated delay case is shown by the dotted line. It can be seen that the LPV controller successfully regulates the tailpipe air-fuel ratio to a preset reference in the face of the large variable delay in the engine model. Hence, the robustness of the LPV controller to delay estimation errors is indeed satisfactory. It can be seen that the three corresponding time responses are almost overlapping. A more detailed view of the simulation results is shown in Fig. 16. It can be observed that for the underestimated delay case the response speed is a bit faster and there is a small increase in the percentage overshoot. For the overestimated delay case, the response speed is a bit slower. This is as expected because the phase margin of the closed-loop system will increase when the time delay is overestimated.
5 Experimental Results

Finally, the controller was implemented on a Ford F-150 truck with a V8 4.6 l lean burn engine. Figure 17 shows the experimental results for a low speed driving condition, which characterizes the long delay case. It can be seen that the tailpipe air-fuel ratio reaches its reference value and settles in about 8 s using the LPV controller. Highway driving experimental results using the LPV controller are presented in Figs. 18 and 19. Figure 18 shows the
tailpipe air-fuel ratio reference (dashed line), measured tailpipe air-fuel ratio (solid line), and the lean burn control activation flag (dashed-dotted line). The controller is activated when the value of the flag is equal to 1. In addition, the simulated tailpipe air-fuel ratio using the simplified engine model is shown (dotted line). The corresponding engine speed and estimated time delay are shown in Fig. 19. It can be observed that the tailpipe air-fuel ratio starts increasing from 1 and converges to the reference after the controller is activated. The settling time is always less than 10 s despite the engine speed changes. No significant overshoot is observed. Clearly, the LPV controller provides satisfactory performance at both low and high engine speeds. Also, it can be seen that the simulated tailpipe air fuel ratio is very close to the actual measurement. This confirms that the simplified engine model is accurate enough for the tailpipe air-fuel regulation purposes.

6 Conclusion

An LPV controller is designed for precise air-fuel ratio control in lean burn SI engines. The controller is scheduled based on the variable time delay in the feedback loop to accommodate the engine speed and air mass flow variability. A first order simplified gain-scheduled controller that includes polynomial forms of the controller gains as a function of the control loop delay is obtained to satisfy desired transient and steady-state response characteristics. Both simulations and experimental results in a Ford F-150 truck with a V8 4.6 l lean burn engine demonstrate the efficiency of the LPV controller to regulate the tailpipe air-fuel ratio for the full engine operating envelope.

In the present work, a constant set-point regulation of the air-fuel ratio is considered during steady-state lean burn operation at various operating conditions. During transient engine operation, the commanded air-fuel ratio varies based on the engine’s capability of lean operation as a function of engine speed and load. As a result, the air fuel ratio reference can vary very rapidly. Consequently, a single feedback controller is not capable of tracking such kind of fast varying air-fuel ratio reference due to the bandwidth limitation imposed by the time delay. Extensions of the proposed control methodology to address transient lean operation using a two degree-of-freedom control strategy will be examined in future work.

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