On the Depletion Effect in Colloids

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Abstract

We formulate and analyze a physically plausible and mathematically sound model to better understand the phenomenon of clustering in colloids. The term *depletion force* refers to the force (in the Newtonian sense) which is associated with the clustering. Our model is stochastic but derived from a deterministic setup in a Newtonian setting. A mathematical transition from the deterministic dynamics of several large particles and infinitely many small particles to a kinetic description of the stochastic motion of the large particles is available. Assuming that the empirical velocity distribution of the small particles is governed by a probability density, the mean-field force on the large particles can be represented as the negative gradient of a scaled version of that density. The stochastic motion of the large particles can then be described by a system of *correlated* Brownian motions. The scaling in the transition preserves a small parameter, the correlation length. From the limiting kinetic stochastic equations we compute the probability flux rates for the difference in position between two large particles. We show that, for short times, two particles sufficiently close together tend to be attracted to each other. This agrees with the *depletion* phenomena observed in colloids. To quantify this effect, we extend the notion of van Kampen's one-dimensional probability flux rate in an appropriate way to account for higher dimensional effects.

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