

LOAD CAPACITY OF BODIES

REUVEN SEGEV

*Department of Mechanical Engineering
Ben-Gurion University Beer-Sheva, Israel, rsegev@bgu.ac.il*

Consider a homogeneous isotropic elastic–perfectly plastic body Ω modeled by a bounded open subset of \mathbb{R}^3 having a smooth boundary $\partial\Omega$. The body is assumed to be supported on an open subset Γ_0 of its boundary and let t be the external surface traction acting on the complementary part, Γ_t , of the boundary. Body forces may be included in the analysis but for the sake of simplicity we omit them here. We prove that there exists a maximal positive number C , to which we refer as the *load capacity ratio*, such that the body will not collapse under any external traction field t bounded by Y_0C , where Y_0 is the yield stress. Thus, while the limit analysis factor of the theory of plasticity pertains to a specific distribution of external loading, the load capacity ratio is independent of the distribution of external loading and implies that no collapse will occur for any field t on $\partial\Omega$ as long as

$$\operatorname{ess\,sup}_{y \in \partial\Omega} |t(y)| < Y_0C.$$

Collapse will occur for some t for which the essential supremum of its magnitude over Γ_t is greater than Y_0C .

The load capacity ratio depends only on the geometry of the body and we prove, see [1], that it is given by

$$\frac{1}{C} = \sup_{w \in LD(\Omega)_D} \frac{\int_{\Gamma_t} |zw| \, dA}{\int_{\Omega} |\varepsilon(w)| \, dV} = \|\gamma_D\|.$$

Here, $LD(\Omega)_D$ is the space of incompressible integrable vector fields w that satisfy the boundary conditions on Γ_0 and for which the corresponding stretchings, or linear strains, $\varepsilon(w)$ are assumed to be integrable. This vector space is equipped with the norm

$$\|w\| = \int_{\Omega} |\varepsilon(w)| \, dV$$

making it into a Banach space. The norm $|\varepsilon(w)|$ on the space of incompressible, or zero-trace, strain matrices should be chosen as the dual of a norm $|\sigma(x)|$ on the space of deviatoric stress matrices induced by the yield criterion, e.g., the Frobenius norm for von-Mises yield criterion. In addition, $\gamma_D : LD(\Omega)_D \rightarrow L^1(\partial\Omega, \mathbb{R}^3)$ is the trace mapping assigning the boundary value $\gamma_D(w)$ to any $w \in LD(\Omega)_D$. It can be shown, see [2], that the trace mapping is well defined. Thus, $1/C$ is the operator norm of the trace mapping.

In addition to these theoretical results we will review some applications of this approach to stress analysis of structures.

REFERENCES

- [1] Segev, R., Load Capacity of Bodies, *International Journal of Non-Linear Mechanics*, **42**, 250–257 (2007).
- [2] Temam, R., *Mathematical Problems in Plasticity*, (a translation of *Problemes mathematiques en plasticite*, Bordas, Paris, 1983) Gauthier-Villars, Paris (1985).