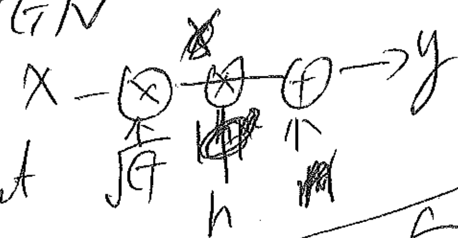
Chapter 6 Performance of Digital Modulation over Wireless channels

6.1 AWGN



without fading

h

$$SNR = \frac{P_r}{N_0 B} = \frac{E_s}{N_0 B T_s} = \frac{E_b}{N_0 B T_b}$$

P_r: received power

B: Bandwidth

$$P_r = P_t \cdot G \cdot |h|$$

G: large propagation gain

$|h|$: fading

$\gamma_s = E_s/N_0$, $\gamma_b = E_b/N_0$, $\gamma_b \triangleq \frac{\gamma_s}{\log_2 M}$

$$SINR = \frac{P_r}{N_0 B + P_I} \qquad P_b \approx \frac{P_s}{\log_2 M}$$

γ_s

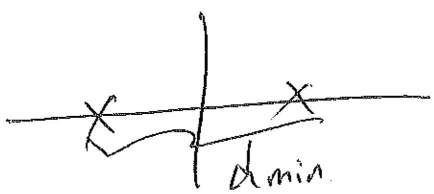
M-aray modulation

P_s: symbol error rate

P_b: bit error rate

6.1.2 Error Probability for BPSK and QPSK

$$BPSK: \quad P_b = Q\left(\frac{d_{min}}{\sqrt{2N_0}}\right) = Q\left(\frac{2\sqrt{E_b}}{\sqrt{2N_0}}\right) = Q\left(\sqrt{\frac{2E_b}{N_0}}\right) = Q\left(\sqrt{2\gamma_b}\right)$$



$$QPSK \quad P_s = 1 - \left[1 - Q\left(\sqrt{2\gamma_b}\right)\right]^2$$

two independent BPSK

$$\approx 2Q\left(\sqrt{\gamma_s/2}\right)$$

EX 6.1

6.1.3 Error Probability for MPSK

$$P_s \approx 2Q\left(\sqrt{2\gamma_s}\sin(\pi/M)\right)$$

example 6.2

6.1.4 Error probability for MPAM and MQAM

average ~~symb~~ power per symbol ~~for MQAM~~

$$\bar{E}_s = \frac{1}{M}\sum_{i=1}^{M} A_i^2 = \frac{1}{3}(M^2-1)d^2$$

For MPAM

$$P_s = \frac{2(M-1)}{M} Q\left(\sqrt{\frac{6\bar{\gamma}_s}{M^2-1}}\right)$$

For MQAM

$$P_s = \frac{2(\sqrt{M}-1)}{\sqrt{M}} Q\left(\sqrt{\frac{3\bar{\gamma}_s}{M-1}}\right)$$

~~Ex~~ Ex 6.3.

6.1.5 error for FSK

BFSK $\quad P_b = Q(\sqrt{\gamma_b})$

MFSK $\quad P_s = \sum_{m=1}^{M} (-1)^{m+1} \binom{M-1}{m} \frac{1}{m+1} \exp\left(\frac{-m\gamma_s}{m+1}\right)$

~~6.~~ 6.1.6 coherent Modulation

Table 6.1

6.1.7 ~~E~~rror probability for Differential modulation

DPSK $P_b = \frac{1}{2} e^{-\gamma_b}$

DQPSK, (6.41)

6.2 Alternative Q Function Representation

$$Q(z) = P(X \geq z) = \int_z^\infty \frac{1}{\sqrt{2\pi}} e^{-x^2/2} dx$$

$$= \frac{1}{\pi} \int_0^{\pi/2} \exp\left[\frac{-z^2}{2\sin^2\phi}\right] d\phi, \quad z > 0$$

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6.3 Fading

~~for~~ average ~~over~~ AWGN over fading Random variable

6.3.1 outage Probability
simple than BER

$$P_{out} = P(\gamma_s < \gamma_0) = \int_0^{\gamma_0} P_{\gamma_s}(\gamma) d\gamma$$

Rayleigh $P_{out} = \int_0^{\gamma_0} \frac{1}{\bar{\gamma}_s} e^{-\gamma_s/\bar{\gamma}_s} d\gamma_s = 1 - e^{-\gamma_0/\bar{\gamma}_s}$

Ex 6.4

6.3.2 Average Probability of Error

$$\bar{P}_s = \int_0^\infty P_s(\gamma) P_{\gamma_s}(\gamma) d\gamma$$

BPSK $\overline{P}_b = \frac{1}{2}\left[1-\sqrt{\frac{\overline{\gamma}_b}{1+\overline{\gamma}_b}}\right] \approx \frac{1}{4\overline{\gamma}_b}$

BFSK $\overline{P}_b = \frac{1}{2}\left[1-\sqrt{\frac{\overline{\gamma}_b}{2+\overline{\gamma}_b}}\right] \approx \frac{1}{4\overline{\gamma}_b}$

Figure 6.1, 6.2, 6.3

DPSK $\overline{P}_b = \frac{1}{2(1+\overline{\gamma}_b)} \approx \frac{1}{2\overline{\gamma}_b}$

6.3.3 Moment Generation Function (MGF) Approach

r.v. pdf $P_\gamma(\gamma)$, $\gamma \geq 0$

MGF: $M_\gamma(s) = \int_0^\infty P_\gamma(\gamma) e^{s\gamma} d\gamma$

"Fourier Transform" for r.v.

Rayleigh: $M_{\gamma_s}(s) = (1 - s\overline{\gamma}_s)^{-1}$

Ricean $M_{\gamma_s}(s) = \frac{1+K}{1+K-s\overline{\gamma}_s} \exp\left[\frac{K s \overline{\gamma}_s}{1+K-s\overline{\gamma}_s}\right]$

Nakagami: $M_\gamma(s) = \left(1 - \frac{s\overline{\gamma}_s}{m}\right)^{-m}$

$$E[\gamma^n] = \frac{\partial^n}{\partial s^n} [M_{\gamma_s}(s)] \Big|_{s=0}$$

For AWGN $P_s = C_1 \exp[-C_2 \gamma_s]$

or $P_s = \int_A^B C_1 \exp[-C_2(x)\gamma] dx$

For fading $\overline{P}_s = \int_0^\infty C_1 \exp[-C_2\gamma] P_{\gamma_s}(\gamma) d\gamma$

$$= C_1 M_{\gamma_s}(-C_2)$$

or $\overline{P}_s = C_1 \int_A^B M_{\gamma_s}(-C_2(x)) dx$

MPSK $P_s(\gamma_s) = \alpha Q(\sqrt{2g\gamma_s}) = \frac{\alpha}{\pi}\int_0^{\pi/2} \exp\left(\frac{-g\gamma}{\sin^2\phi}\right) d\phi$

~~MPSK~~ $\bar{P}_s = \frac{\alpha}{\pi}\int_0^{\pi/2} M_{\gamma_s}\left(\frac{-g}{\sin^2\phi}\right) d\phi.$

Ex 6.5.

MQAM $P_s(\gamma_s) = \cancel{\#}$ (6.80)

$\bar{P}_s = (6.8.1) - (6.8.4)$

6.3.4 Combined outage and Average Error probability

log normal ~~s~~hawdowing + Rayleigh fading

- $\bar{\bar{\gamma}}_s$ average both
- $\bar{\gamma}_s$ average fading
- γ_s instantaneous

6.4 Doppler spread

$\bar{P}_b = \frac{1}{2}\left[\frac{1+\bar{\gamma}_b(1-\rho_c)}{1+\bar{\gamma}_b}\right)$ ρ_c is channel correlation coefficient

$\boxed{\text{error floor}}$

~~Ex 6.7~~ Fig 6.4 ~~x~~

6.5 Intersymbol interference

$\gamma_s = \frac{P_r}{N_0 B + I}$

Power control