Carrier Modeling

Hydrogen atom

\[ E_H = \frac{m_0 \cdot q^4}{2 (4\pi\varepsilon_0 + \hbar n)^2} = -\frac{13.6 \text{ eV}}{n^2} \]

\( n = 1, 2, 3, \ldots \)

1 eV (electron volt) = 1.6 \times 10^{-19} \text{ Joules}

\text{energy an electron gains passing a potential difference of 1 Volt}.
Eletrons in Semiconductors

in Si: 14 electrons per atom

$5 \times 10^{22}$ atoms/cm$^3$

It is unrealistic to track them all.

Si: $\frac{3s^2 \ 3p^2}{4 \text{electrons}}$

-- Line represents shared valence electron

-- Diagram showing energy levels and electron transitions

- $4N$ empty states
- $2N+2N$ filled states
Energy band diagram

$E_G = E_C - E_v$

No carriers

The electron

The hole

Both electrons and holes participate in conduction
Band Gap and Material Classification

(a) Insulator

\[ E_g \approx 8.0 \text{eV (SiO}_2\text{)} \]
\[ E_g \approx 5.2 \text{eV (Diamond)} \]

(b) Semiconductor

\[ E_g = 1.42 \text{eV (GaAs)} \]
\[ E_g = 1.12 \text{eV (Si)} \]
\[ E_g = 0.66 \text{eV (Ge)} \]
(Clear room temperature)

Very narrow \( E_c \) or \( E_v \) over \( E_v \) or \( E_c \)

metal
Carrier Properties

Newton's law:

\[ F = -q \cdot E = m_0 \frac{d \vec{v}}{dt} \]

Charge: \( q \)

Electric field: \( E \)

Diagram of force and electric field.

In crystal:

\[ F = -q \cdot E = m^* \frac{d \vec{v}}{dt} \]

For electrons:

For holes: the same equation

\[ m^*_n \rightarrow m^*_p \]

\[ -q \rightarrow q \]

Can treat electrons and holes as "QUASI"-particles and use classical treatments.
<table>
<thead>
<tr>
<th>Material</th>
<th>$m_n^* / m_0$</th>
<th>$m_p^* / m_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Si</td>
<td>1.18</td>
<td>0.81</td>
</tr>
<tr>
<td>Ge</td>
<td>0.55</td>
<td>0.36</td>
</tr>
<tr>
<td>GaAs</td>
<td>0.066</td>
<td>0.52</td>
</tr>
</tbody>
</table>

**Intrinsic Carrier Concentration**

$n = \text{number of electrons} / \text{cm}^3$

$p = \text{number of holes} / \text{cm}^3$

in equilibrium $n = p = N_i$

\[
\begin{align*}
E_g = 1.42 \text{eV} & \quad N_i \approx 2 \times 10^6 / \text{cm}^3 \quad \text{GaAs} \\
E_g = 1.12 \text{eV} & \quad N_i \approx 1 \times 10^{10} / \text{cm}^3 \quad \text{Si} \\
E_g = 0.66 \text{eV} & \quad N_i \approx 2 \times 10^{13} / \text{cm}^3 \quad \text{Ge}
\end{align*}
\]
In Si: \[ 5 \times 10^{22} \text{ atoms } / \text{cm}^3 \]
\[ 2 \times 10^{23} \text{ bonds } / \text{cm}^3 \]
\[ n_i \sim 10^{10} / \text{cm}^3 \rightarrow 1 \text{ bond / per } 10^{13} \text{ bonds is broken at room temperature.} \]

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**Doping**

\[ \begin{array}{c}
\vdots \text{P} \vdots \\
O = O = O = O \\
\vdots \text{B} \vdots \\
\vdots \end{array} \]

P has 5 electrons to share

"Donor"

\[ \begin{array}{c}
\vdots \text{B} \vdots \\
\vdots \text{P} \vdots \\
\vdots \end{array} \]

B has 3 electrons to share

"Acceptor"
Column V

P
As
Sb \{ donors

Column VI

B
Ga \{ acceptors
In
Al

\[ E_B = - \frac{m^* g^4}{2 (4\pi k_b T)^2} = \]

\[ = \frac{m^*}{m_0} \frac{1}{k_b^2} E_{H_{n=1}} \approx -0.1 \text{ eV} \]

\( (k_b = 11.8 \text{ in Si}) \)

Dopant binding energies:

| Donor | \( |E_B| \)  | Acceptor | \( |E_B| \) |
|-------|---------|----------|---------|
| Sb    | 0.039 eV| B        | 0.045 eV|
| P     | 0.045 eV| Al       | 0.067 eV|
| As    | 0.054 eV| Ga       | 0.072 eV|
|       |         | In       | 0.16 eV |
Donors and Acceptors are used to tailor carrier concentration in semiconductor materials.
Density of States

\[ g_c(E) = \frac{m_n^* \sqrt{2m_n^*(E-E_c)}}{T^2 + \hbar^2} \]

\[ g_v(E) = \frac{m_p^* \sqrt{2m_p^*(E_v-E)}}{T^2 + \hbar^2} \]

\[ g_c(E) \, dE \rightarrow \text{number of conduction band states / cm}^3 \text{ in the energy range between } E \text{ and } E + dE \]
Fermi Junction

\[ f(E) = \frac{1}{1 + e^{(E-E_F)/kT}} \rightarrow \text{probability of having an electron occupy energy level } E, \]

\[ E_F = \text{Fermi energy/level} \]
\[ k = \text{Boltzmann constant} \quad (= 8.617 \times 10^{-5} \text{eV}/K) \]

\[ f(E) = \begin{cases} \frac{1}{2} & \text{if } E = E_F \\ \frac{1}{2} + \text{as } E \rightarrow \infty & \text{if } E \geq E_F + 3kT \\ 1 - e^{-(E-E_F)/kT} & \text{if } E \leq E_F - 3kT \end{cases} \]

At room temperature \((T = 300K)\)
\[ kT = 0.0259 \text{ eV}, \quad 3kT = 0.0777 \text{ eV} \]