Exercise

Probability of a state filled at the conduction band edge ($E_C$) is equal to the probability that a state is empty at the valence band edge ($E_V$). Where is the Fermi level? [Intrinsic material]

$f(E_c) \Rightarrow$ probability of electron occupying states at $E_c$

$1 - f(E_v) \Rightarrow$ probability of not occupied state at $E_v$

$f(E_c) = 1 - f(E_v)$

$$\frac{1}{1 + e^{\frac{(E_c-E_F)/kT}{1 + e^{\frac{(E_v-E_F)/kT}}}} = 1 - \frac{1}{1 + e^{\frac{(E_v-E_F)/kT}} = \frac{1}{1 + e^{\frac{E_F-E_v}{kT}}}}$$

$$\frac{E_c - E_F}{kT} = \frac{E_F - E_v}{kT}$$

$$\Rightarrow$$

$$E_F = \frac{E_c + E_v}{2} \leq \text{middle of the band gap}$$
Equilibrium Distribution of Carriers

\[ g_c(E) f(E) \] - distribution of electrons in conduction band

\[ g_v(E) [1 - f(E)] \] - distribution of holes (empty states) in the valence band
Equilibrium carrier concentrations

\[ n = \int_{E_c}^{E_{\text{top}}} g_c(E) \, f(E) \, dE \]

\[ g_c(E) \] density of states

\[ f(E) \] probability of a state being occupied

\[ E_r \]

\[ p = \int_{E_{\text{bottom}}}^{E_r} g_v(E) \left[ 1 - f(E) \right] \, dE \]

\[ E_{\text{bottom}} \]

\[ n = \frac{m^*_n}{\hbar^2} \sqrt{2m^*_n} \int_{E_c}^{E_{\text{top}}} \frac{\sqrt{E - E_c}}{1 + e^{(E - E_F)/kT}} \, dE \]

Letting \( \eta = \frac{(E - E_c)}{kT} \)

\[ \eta_c = \frac{E_F - E_c}{kT} \]

\[ E_{\text{top}} = \infty \quad \text{[approximation]} \]
\[ n = \frac{m^* \sqrt{2m^* (kT)^3}}{\pi^2 \hbar^3} \int_0^\infty \frac{y^{1/2} \, dy}{1 + e^{y - y_c}} \]

Tabulated function, Fermi-Dirac integral of order \( 1/2 \)

Define

\[ F_{1/2}(y_c) = \int_0^\infty \frac{y^{1/2} \, dy}{1 + e^{y - y_c}} \]

\[ N_c = 2 \left[ \frac{m^* kT}{2\pi \hbar^2} \right]^{3/2} \quad \text{effective density of conduction band states} \]

\[ N_v = 2 \left[ \frac{m^* kT}{2\pi \hbar^2} \right]^{3/2} \quad \text{effective density of valence band states} \]

\[ n = N_c \frac{2}{\sqrt{\pi}} F_{1/2}(y_c) \]

and

\[ P = N_v = \frac{2}{\sqrt{\pi}} F_{1/2}(y_v), \quad y_v = \frac{E_v - E_F}{kT} \]
at 300 K: \[ N_{c,v} = \left( 2.510 \times 10^{19} \text{ cm}^{-3} \right) \left( \frac{m^*_s p}{m^*_0} \right)^{3/2} \]

Simplified forms:

\[ E_F \leq E_c - 3kT, \text{ then } \frac{1}{1 + e^{-(\eta - \eta_c)}} e^{-(\eta - \eta_c)} \]

\[ F_{1/2}(\eta_c) = \frac{\sqrt{2}}{2} e \]

\[ E_F \geq E_v + 3kT, \text{ then } \frac{(E_v - E_F)}{kT} \]

\[ F_{1/2}(\eta_v) = \frac{\sqrt{2}}{2} e \]

or

\[ E_v + 3kT \leq E_F \leq E_c - 3kT \]

\[ \frac{(E_F - E_c)}{kT} \]

\[ n = N_c e \]

\[ p = N_v e \]

\[ E_F = \text{Degenerate semiconductor} \]

\[ E_v \text{ Non Degenerate semiconductor} \]

\[ E_c \]
In intrinsic semiconductor

\[ E_i = E_c \] lies close to midgap

Hence

\[ n = n_i = \frac{N_c}{N_{intrinsic}} e \]

and

\[ p = n_i = \frac{N_v}{N_{intrinsic}} e \]

\[ \frac{(E_i - E_c) \sqrt{N_c}}{kT} \]

\[ \frac{(E_v - E_i) \sqrt{N_v}}{kT} \]

\[ N_c = n_i \sqrt{N_c} \]

\[ N_v = n_i \sqrt{N_v} \]

Finally,

\[ \frac{(E_F - E_i) \sqrt{N_c}}{kT} \]

\[ n = n_i e \]

\[ \frac{(E_i - E_F) \sqrt{N_c}}{kT} \]

\[ p = n_i e \]
More useful equations:

\[ n_i^2 = \frac{N_c N_v}{e} \left( \frac{(E_c-E_v)}{kT} - \frac{E_g}{2kT} \right) \]

\[ n_i = \sqrt{N_c N_v} e \]

Also,

\[ n^* p = n_i^2 \]

**Charge Neutrality**

\[ \frac{\text{charge}}{\text{cm}^3} = q^* p - q^* n + q^* N_D^+ - q^* N_A^- = 0 \]

\[ p - n + N_D^+ - N_A^- = 0 \]

**or**

\[ p - n + N_D - N_A = 0 \]
Carrier Concentration Calculations

\[ p = \frac{n_i^2}{n} \]

From charge neutrality:

\[ \frac{n_i^2}{n} - n + N_D - N_A = 0 \]

or

\[ n^2 - n (N_D - N_A) - n_i^2 = 0 \]

Solving for \( n \):

\[ n = \frac{N_D - N_A}{2} + \left[ \left( \frac{N_D - N_A}{2} \right)^2 + n_i^2 \right]^{1/2} \]

\[ p = \frac{n_i^2}{n} = \frac{N_A - N_D}{2} + \left[ \left( \frac{N_A - N_D}{2} \right)^2 + n_i^2 \right]^{1/2} \]

Special cases:

1. Intrinsic Semiconductor

\[ N_A = 0, \quad N_D = 0 \]

\[ n = n_i \]

\[ p = n_i \]
(2) Doped Semiconductor where 
\[ N_D - N_A \approx N_D \gg n_i \quad \text{or} \quad N_A - N_D \approx N_A \geq n_i \]

[typically, \( n_i \sim 10^{10} / \text{cm}^3 \), doping \( \sim 10^n \text{ cm}^{-3} \)]

\[ n \approx N_D \quad N_D \gg N_A, \quad N_D \gg n_i \]
\[ p \approx \frac{n_i^2}{N_D} \]

Similarly

\[ p \approx N_A \quad N_A \gg N_D, \quad N_A \gg n_i \]
\[ n \approx \frac{n_i^2}{N_A} \]

(3) Doped Semiconductor at high temperature
\[ n_i \gg \left| N_D - N_A \right| \]

\[ n = p = n_i \]

(4) Compensated
\[ N_D - N_A = 0 \quad n = p = n_i \]

However, \( N_D \) and \( N_A \) need to be retained in all carrier concentration expressions.